Free Vibrations of a Certain Geometrically Nonlinear System with Initial Imperfection

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In this paper, the free vibration of a geometrically nonlinear system with initial imperfection is studied. The equations of motion of a beam are assumed within the framework of the moderately large bending theory. The effects of an initial imperfection and amplitude on the nonlinear vibration frequency and value of axial internal force in a beam are presented. For solving the problem, the Lindstedt-Poincaré method is used.

Nomenclature

A = cross-sectional area of beam
C = spring constant
e = eccentricity of load
E = Young's modulus of beam
$F(t)$ = internal longitudinal force in spring
J = moment of inertia of beam
L = beam length
P = force loading the system
$S(t)$ = internal longitudinal force in beam
S_0 = first approximation of force S
t = time coordinate
$U(x,t)$ = longitudinal displacement of beam
$W(x,t)$ = lateral displacement of beam
W_x = $\partial W(x,t)/\partial x$
β = $cL/(EA)$, nondimensional spring constant
ϵ = small dimensionless parameter ($\epsilon \ll 1$)
ω = nonlinear free frequency
ω_0 = first approximation of nonlinear frequency ω
ρ = mass density

Introduction

THE problems concerning vibration and stability of overbraced frames can be described in terms of moderately large bending theory, in which one takes into account the geometric nonlinearity generated by the transverse displacement of frame beams. In Refs. 1-7, a nonlinear buckling analysis of planar structures subjected to conservative and nonconservative forces has been presented. This analysis was based on a static approach and only the divergence-type instability was studied. Vibration of these systems had not been examined, whereas the problem of nonlinear free vibration of beams with immovable ends received the attention of several authors (cf. Refs. 8-9 for bibliographical information). Both the continuum approach and the finite-element methods were used. Bhashyam and Prathap[8] and Sarma and Varadan[9] interpreted ω^2 not as a frequency but only as a proportionality coefficient between $\ddot{w}$ and w at the point of maximum amplitude of the transverse displacement.

Bhashyam and Prathap[8] referred to ω^2 as the characteristic nonlinear frequency parameter, whereas other scholars use the traditional name: the nonlinear frequency (cf. Ref. 9). In this work, the latter expression is used. The relationship ($\ddot{w} = -\omega^2 w$) is used in the case of clamped and hinged-clamped beams for which the mode shape changes with the increase in the amplitude of vibration. In the case of a change of mode shape, the variables-separable solution is not a true solution. For the simply supported beams, the change of mode shape does not occur and the application of the relationship $\ddot{w}_{\max}(x,t_1) = -\omega^2 w_{\max}(x,t_1)$, $\dot{w}(x,t_1) = 0$, is valid both in the case of the mode-shape change and in the absence of such changes. The change of the mode shape for clamped beams has been observed earlier by Evensen.[10]

For clamped and hinged-clamped beams, Bhashyam and Prathap[8] have compared the values of ω^2 and axial force, obtained by means of the Rayleigh quotient finite-element method (RQFEM) and the Galerkin finite-element formulation (GFEM), with the results obtained by means of the Lindstedt-Poincaré method (L-PM).

Sarma and Varadan[9] have compared additionally the above results with those obtained by means of the Lagrange type of formulation for FEM. The Lagrange FEM (L-FEM) results agree very well with those of the GFEM, whereas the RQFEM results agree with the L-PM results. All of these results agree very well with each other at low levels of amplitude. One should notice also that the agreement of data for higher levels of amplitude is reasonably good.

For literature quoted above, it is evident that GFEM, L-FEM, RQFEM, and L-PM are equivalent in studying nonlinear vibrations of beams described by the equations of moderately large bending theory.

The present work deals with the analysis of the free vibrations of a geometrically nonlinear continuous system characterized by an initial imperfection. For solving the problem, the Lindstedt-Poincaré method is used, as in Refs. 11 and 12. Our principal purpose is the analysis of the effect of an initial imperfection on the free vibration of the system. This system is presented in Fig. 1. As can be seen, it is composed of a simply supported beam. One end of this beam ($x = L$) is supported by spring (2) in the longitudinal direction. From the equality of longitudinal displacements of the beam and spring, the geometrical nonlinearity of this system results. The initial imperfection is produced by eccentricity e, on which the conservative forces act.

Mathematical Formulation

According to the moderately large bending theory, the equilibrium equations for Hookean material are in the following forms:

In the transverse direction,

$$EJ\,W_{,xxxx} - \left[EA\left(U_{,x} + \tfrac{1}{2}W_{,x}^2\right)W_{,x}\right]_{,x} + \rho A W_{,tt} = 0 \quad (1)$$

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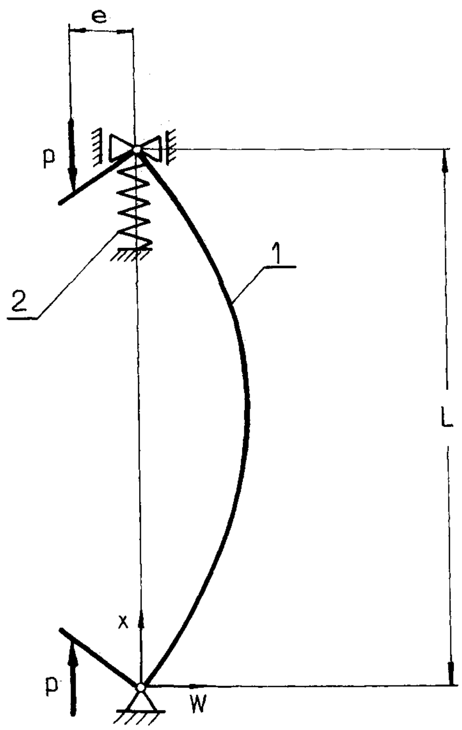

Fig. 1 Scheme of a system.

In the in-plane direction,

$$S = -\left[EA\left(U_{,x} + \tfrac{1}{2}(W_{,x})^2\right)\right] \tag{2}$$

The following dimensionless quantities are introduced:

$$\xi = \frac{x}{L}, \qquad \tau = \omega t, \qquad u = \frac{U}{L}, \qquad w = \frac{W}{L},$$

$$p = \frac{eLP}{EJ}, \qquad s = \frac{L^2 S}{EJ} \tag{3}$$

Equations (1) and (2) in the dimensionless coordinates are in the form

$$w_{,\xi\xi\xi\xi} + s w_{,\xi\xi} + r\omega^2 w_{,\tau\tau} = 0 \tag{4}$$

where

$$r = \frac{\rho A L^4}{EJ}, \qquad s = -\left[\frac{AL^2}{J}\left(u_{,\xi} + \tfrac{1}{2}(w_{,\xi})^2\right)\right] \tag{5}$$

The dimensionless lateral displacements of the beam satisfy the conditions (Fig. 1):

$$w|_{\xi=0} = w|_{\xi=1} = 0 \tag{6}$$

$$w_{,\xi\xi|\xi=0} = w_{,\xi\xi|\xi=1} = -p \tag{7}$$

On the basis of the equality condition of longitudinal beam and spring displacements for $\xi = 1$ and Eq. (5), we obtain

$$s(\tau) = a - \tfrac{1}{2} b \int_0^1 (w_{,\xi})^2 \mathrm{d}\xi \tag{8}$$

where

$$a = \frac{PL^2}{EJ}\frac{1}{1+\beta}, \qquad b = \frac{AL^2}{J}\frac{\beta}{1+\beta}$$

Application of the Perturbation Method to the System

In order to determine the solution, the method of small parameter, which is also known as the method of strained parameters or the Lindstedt-Poincaré method, is applied.[11,12] The functions $w(\xi,\tau)$ and ω^2 are expanded into the Taylor series with respect to the parameter ϵ in the surrounding $\epsilon = 0$, i.e., we assume

$$w(\xi,\tau,\epsilon) = \sum_{n=0}^{N} \frac{\epsilon^n}{n!} w_n(\xi,\tau) + \mathcal{O}(\epsilon^{N+1}) \tag{9}$$

$$\omega^2(\epsilon) = \sum_{n=0}^{N} \frac{\epsilon^n}{n!} \omega_n^2 + \mathcal{O}(\epsilon^{N+1}) \tag{10}$$

Moreover, owing to the relationships given by Eqs. (8) and (9), we have

$$s(\tau,\epsilon) = \sum_{n=0}^{N} \frac{\epsilon^n}{n!} s_n(\tau) + \mathcal{O}(\epsilon^{N+1}) \tag{11}$$

where

$$s_0 = a + \tfrac{1}{2} I_{0,0} \tag{12}$$

$$s_n = \tfrac{1}{2} \sum_{k=0}^{n} \binom{n}{k} I_{k,n-k}, \qquad n = 1,2,\ldots,N \tag{13}$$

with

$$I_{i,j} = -b \int_0^1 \frac{\partial w_i}{\partial \xi}\frac{\partial w_j}{\partial \xi} \mathrm{d}\xi \tag{14}$$

By considering expansions of Eqs. (9–11) in Eq. (4) and grouping the terms of the same powers ϵ, we obtain the following equations (dots denote a derivative with respect to τ):

$$\epsilon^0{:}\; w_0^{\mathrm{IV}} + s_0 w'' + r\omega_0^2 \ddot{w}_0 = 0 \tag{15}$$

$$\epsilon^1{:}\; L(w_n) = -r\omega_1^2 \ddot{w}_0 \tag{16}$$

$$\epsilon^n{:}\; L(w_n) = -\sum_{k=1}^{n-1} \binom{n}{k} \left[s_k w''_{n-k} + \tfrac{1}{2} I_{k,n-k} w''_0 + r\omega_k^2 \ddot{w}_{n-k}\right] - r\omega_n^2 \ddot{w}_0, \qquad n = 2,3,\ldots,N \tag{17}$$

where

$$L(w_k) \equiv w_k^{\mathrm{IV}} + s_0 w''_k + r\omega_0^2 \ddot{w}_k + I_{0,k} w''_0, \qquad k = 1,\ldots,N \tag{18}$$

In order to determine the solutions of Eqs. (15–17), it is assumed that the unknown functions $w_k(\xi,\tau)$ are the periodical functions in the form

$$w_k(\xi,\tau) = \sum_{i=0}^{k} w_k^i(\xi) \cos(i\tau) \tag{19}$$

Furthermore, on the basis of Eqs. (12–14) and Eq. (19), we also have

$$s_k(\tau) = \sum_{i=0}^{k} s_k^i \cos(i\tau) \tag{20}$$

The following boundary conditions result from Eqs. (6) and (7) for the functions

$$w_k^i(0) = w_k^i(1) = 0, \qquad k = 0,1,\ldots,N, \quad i = 0,1,\ldots,k \tag{21}$$

$$\left(w_k^i\right)''|_{\xi=0} = \left(w_k^i\right)|_{\xi=1} = 0, \qquad k = 0,1,\ldots,N, \quad i = 0,1,\ldots,k \tag{22}$$

$$\left(w_0^0\right)''|_{\xi=0} = \left(w_0^0\right)''|_{\xi=1} = -p \tag{23}$$

Because of Eq. (19), we obtain

$$w_0(\xi,\tau) \equiv w_0^0(\xi) \tag{24}$$

This means that the term $\ddot{w}_0$ nullifies in Eqs. (15–17).

Solution of the Problem

Equation (15), after considering Eq. (24), becomes a governing differential equation of the equilibrium of a beam. This equation and the conditions given by Eqs. (21) and (23) are therefore satisfied by the function:

$$w_0^0(\xi) = \frac{p}{s_0}\left(\frac{\cos k_s(\xi - ½)}{\cos \dfrac{k_s}{2}} - 1\right) \tag{25}$$

where $k_s^2 = s_0$. By substituting Eq. (25) into Eq. (12), one gets the transcendental equation for s_0:

$$a - s_0 + \frac{bp^2}{4s_0 \cos^2 \dfrac{k_s}{2}}\left(\frac{\sin k_s}{k_s} - 1\right) = 0 \tag{26}$$

The functions $w_1^i(\xi)$, $i = 0,1$, satisfy the equation which is obtained by substituting Eq. (19) into Eq. (16) for $k = 1$:

$$L_0(w_1^i) \equiv (w_1^i)^{IV} + s_0(w_1^i)'' + s_1^i w_0'' = ir\omega_0^2 w_1^i, \qquad i = 0,1 \tag{27}$$

Homogeneous differential Eq. (27) with homogeneous boundary conditions, Eqs. (21) and (22), form a class of problems known as eigenvalue problems. One may show that for $s_0 < \pi^2 (S_0 < \pi^2 EJ/L^2)$ and $i = 0$,

$$w_1^0(\xi) \equiv 0 \tag{28}$$

The solution of Eq. (27) for $i = 1$ with conditions given by Eqs. (21) and (22) ($k = 1$) is obtained in the form

$$w_1^1(\xi) = Ds_1^1\left[\frac{\cos k_s(\xi - ½)}{\cos \dfrac{k_s}{2}} - \frac{1}{\alpha_1^2 + \alpha_2^2}\right.$$

$$\left.\times\left(\alpha_1^2 \frac{\cosh \alpha_1(\xi - ½)}{\cosh \dfrac{\alpha_1}{2}} + \alpha_2^2 \frac{\cos \alpha_2(\xi - ½)}{\cos \dfrac{\alpha_2}{2}}\right)\right] \tag{29}$$

where

$$D = \frac{pL}{r\omega_0^2}, \quad \alpha_i^2 = ½\left\{(-1)^i s_0 + \left[(s_0)^2 + 4r\omega_0^2\right]^{½}\right\}, \quad i = 1,2$$

In the solution given by Eq. (29), s_1^1 is an arbitrary constant which is determined from the normalization condition:

$$W_1^1\left(\frac{L}{2}\right) = 1 \tag{30}$$

From Eq. (13) for $n = 1$, taking into account Eqs. (19) and (20), the following equation of the frequency ω_0 is obtained:

$$s_1^1 = I_{1,0} \tag{31}$$

By substituting Eq. (29) and using the normalization condition of Eq. (30), Eq. (31) takes the form

$$bDp\left\{½\cos^{-2}\frac{k_s}{2}\left(1 - \frac{\sin k_s}{k_s}\right) - \frac{2}{\alpha_1^2 + \alpha_2^2}\right.$$

$$\times\left[\frac{\alpha_1^2}{\alpha_2^2}\left(\alpha_1 \tanh \frac{\alpha_1}{2} + k_s \tan \frac{k_s}{2}\right)\right.$$

$$\left.\left.+\frac{\alpha_2^2}{\alpha_1^2}\left(\alpha_2 \tan \frac{\alpha_2}{2} - k_s \tan \frac{k_s}{2}\right)\right]\right\} = -1 \tag{32}$$

On the left-hand side of Eqs. (16) and (17), the same operator occurs. Since homogeneous Eq. (16), after considering Eq. (28), has a nonzero solution, the orthogonality condition (cf. Ref. 1) should be satisfied so that the nonzero solutions of Eq. (17) might exist. This condition may be presented as follows:

$$\sum_{k=1}^{n-1}\binom{n}{k}\int_0^{2\pi}\int_0^1 w_1\left[s_k\frac{\partial^2 w_{n-k}}{\partial\xi^2} + \frac{1}{2}I_{k,n-k}\frac{\partial^2 w_0}{\partial\xi^2} + r\ddot{w}_{n-k}\,\omega_k^2\right]$$

$$\times\, d\xi\, d\tau = 0, \qquad n = 2,\ldots,N \tag{33}$$

From the orthogonality condition of Eq. (33), the unknown values of ω_j^2, $j = 1,2,\ldots,N-1$, are determined, and for $n = 2$, one gets

$$\omega_1^2 = 0 \tag{34}$$

whereas for $n = 3$, we have

$$\omega_2^2 = \frac{\left[\displaystyle\int_0^{2\pi}\left(s_1 I_{1,2} + \frac{1}{2}s_2 I_{1,1}\right)d\tau\right]}{\left[br\displaystyle\int_0^{2\pi}\int_0^1 (w_1)^2\, d\xi\, d\tau\right]} \tag{35}$$

By substituting Eqs. (19) and (20) in Eq. (17), we get the differential equations for components w_i^j, $i = 2,\ldots,N$, $j = 0,1,\ldots,i$. The solutions of these equations for $n = 2$ at zero values of the boundary conditions given by Eqs. (21) and (22) are as below:

$$w_2^0(\xi) = \left(D\, s_1^1\right)^2\left[A_1 + \frac{\cos k_s(\xi - ½)}{\cos \dfrac{k_s}{2}}A_2 - \frac{k_s}{2\cos \dfrac{k_s}{2}}\right.$$

$$\times\left(\frac{\sin k_s\xi}{2\cos \dfrac{k_s}{2}} - \xi \sin k_s(\xi - ½)\right)G_0 + \frac{\cosh \alpha_1(\xi - ½)}{\cosh \dfrac{\alpha_1}{2}}G_1$$

$$\left.+\frac{\cos \alpha_2(\xi - ½)}{\cos \dfrac{\alpha_2}{2}}G_2\right] \tag{36}$$

$$w_2^1(\xi) = w_1^1(\xi) \tag{37}$$

$$w_2^2(\xi) = \left(D\, s_1^1\right)^2 \left[\frac{\cosh \beta_1 (\xi - 1/2)}{\cosh \frac{\beta_1}{2}} B_1 + \frac{\cos \beta_2 (\xi - 1/2)}{\cos \frac{\beta_2}{2}} B_2 - \frac{\cos k_s (\xi - 1/2)}{\cos \frac{k_s}{2}} H_0 + \frac{\cosh \alpha_1(\xi - 1/2)}{\cosh \frac{\alpha_1}{2}} H_1 + \frac{\cos \alpha_2(\xi - 1/2)}{\cos \frac{\alpha_2}{2}} H_2 \right] \qquad (38)$$

where

$$A_1 = \frac{p\, s_2^0}{(D\, s_0\, s_1^1)^2}, \qquad A_2 = -\frac{L\, s_0}{D\, r\, \omega_0^2} - A_1$$

$$G_0 = \frac{1}{LDs_0} - A_1, \qquad G_i = \frac{(-1)^i\, L \alpha_i^2}{EJ\, D\, \alpha_{3-i}^2\, (\alpha_1^2 + \alpha_2^2)}$$

$$B_i = \frac{L}{(Ds_1^1)^2\, r\omega_0^2(\alpha_1^2 + \alpha_2^2)} \left[1/3 \left(1/4\, s_0 \beta_i^2 + (-1)^i\, r\omega_0^2 \right) D\, (s_1^1)^2 + 1/4\, Lps_2^2\beta_2^2 \right]$$

$$H_0 = \frac{Ls_0}{4\, r\, (Ds_1^1)^2} \left[D\, (s_1^1)^2 - Lp \frac{s_2^2}{s_0} \right], \qquad H_i = (-1)^{i+1} \frac{D\, (s_1^1)^2\, \alpha_i^4}{3r\, (\alpha_1^2 + \alpha_2^2)}$$

$$\beta_i^2 = 1/2 \left\{ (-1)^i\, s_0 + \left[(s_0)^2 + 16\, r\, \omega_0^2 \right]^{1/2} \right\}, \qquad i = 1,2$$

The components s_2^i, $i = 0,1,2$, determined on the basis of Eq. (13) and Eqs. (19-20) are in the following form:

$$s_2^0 = -b \left[\int_0^1 (w_0^0)'\, (w_2^0)'\, d\xi + 1/2 \int_0^1 \left((w_1^1)' \right)^2 d\xi \right] \qquad (39)$$

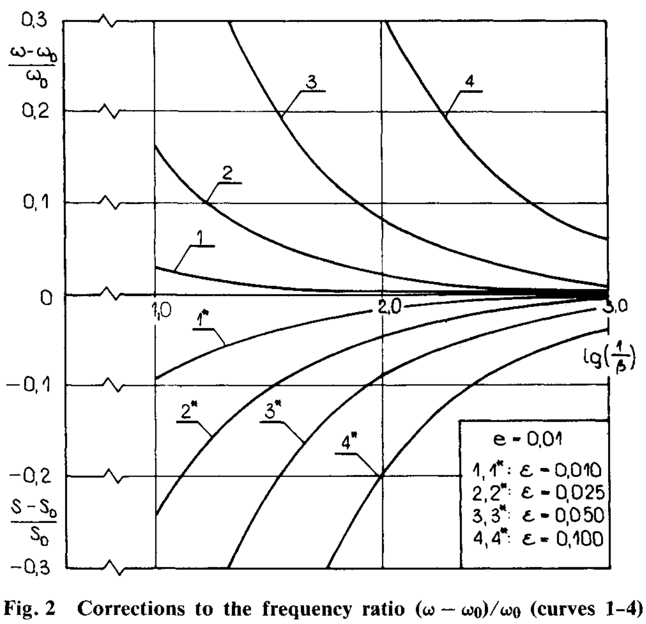

Fig. 2 Corrections to the frequency ratio $(\omega - \omega_0)/\omega_0$ (curves 1-4) and to the internal force amplitude ratio $(S - S_0)/S_0$ (curves 1*-4*) for various β and ϵ (first mode).

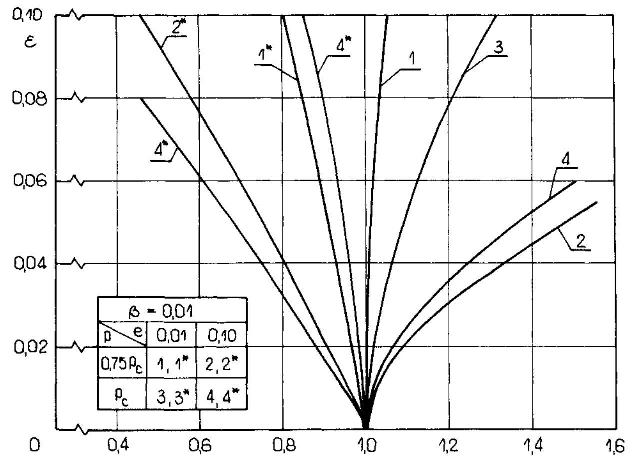

Fig. 3 Variations of nonlinear frequency ratio ω/ω_0 (curves 1-4) and axial force ratio S/S_0 (curves 1*-4*) for various ϵ (first mode).

$$s_2^1 = -b \int_0^1 (w_0^0)'(w_2^1)'\, d\xi \qquad (40)$$

$$s_2^2 = -b \left[\int_0^1 (w_0^0)'\, (w_2^2)'\, d\xi + 1/2 \int_0^1 \left((w_1^1)' \right)^2 d\xi \right] \qquad (41)$$

The solutions of $w_0^0(\xi)$, $w_1^1(\xi)$, $w_2^0(\xi)$, $w_2^1(\xi)$, and $w_2^2(\xi)$ [respectively, Eqs. (25, 29, 36-38)] obtained earlier are substituted into Eqs. (39-41). After some algebra, one gets the formulae expressing s_2^0, s_2^1, and s_2^2. The values of component forces s_2^0, s_2^1, and s_2^2 are calculated directly from the expressions obtained.

Solution for $e = 0$

The solution of the problem in the case of loading the system by an axial force ($e = 0$) is obtained on the basis of the equations given in the preceding section by changing the condition [Eq. (7)] and consequently Eq. (23) into a homogeneous condition ($p = 0$). For the force $S_0^* \epsilon \langle 0, \pi^2 EJ/L^2 \rangle$, there exists the rectilinear equilibrium form $[w_0(\xi) \equiv 0]$. For $S_0^* = \pi^2 EJ/L^2$, we have the curvilinear form of the bar equilibrium $[w_o(\xi) \neq 0]$. Therefore, depending on the value of the force S_0^*, there are two different solutions. By considering the course of calculation given for a longitudinally deflected system ($e \neq 0$), the solution for $e = 0$ is

For $0 \le S_0^* < \dfrac{\pi^2 EJ}{L^2}$,

$$w_0^0 = w_2^0 = w_2^2 = 0, \quad w_1^1 = w_2^1 = \sin\pi\xi, \quad \omega_0^* = \left(\frac{\pi}{L}\right)^2 B \sqrt{\frac{LB}{\rho A}}$$

For $S_0^* = \dfrac{\pi^2 EJ}{L^2}$,

$$w_0^0 = B \sin \pi\xi, \qquad w_1^1 = w_2^1 = \sin\pi\xi, \qquad w_2^0 = -\frac{3}{2BL} \sin \pi\xi,$$

$$w_2^2 = \frac{1}{2BL} \sin \pi\xi$$

where

$$B^2 = \left(\frac{2}{L}\right)^2 \frac{1}{EA\beta} \left[P - \frac{\pi^2 EJ}{L^2} (1 + \beta) \right]$$

Results of the Numerical Calculation

The calculation is performed for the following fundamental physical and geometrical data: $E = 200.0$ GPa, $J = 2.042 \times$

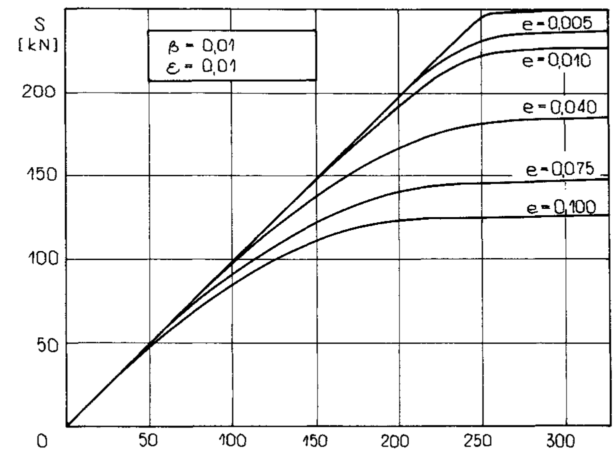

Fig. 4a Axial internal force S in a beam vs loading force P for various e at $\beta = 0.001$ and $\epsilon = 0.01$ (first mode).

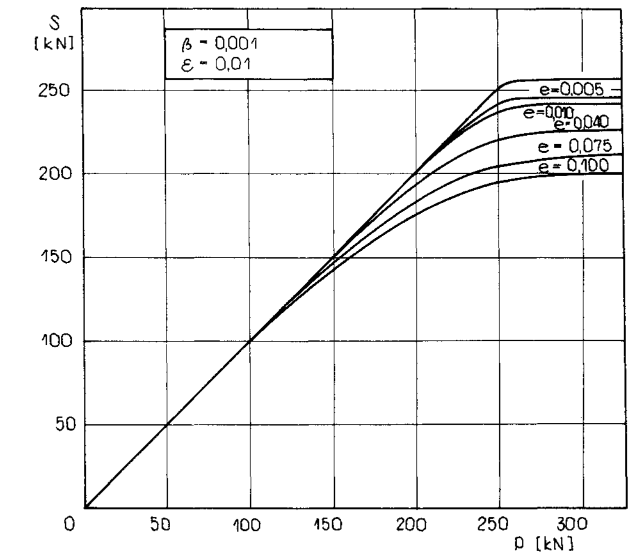

Fig. 4b Axial internal force S in a beam vs loading force P for various e at $\beta = 0.001$ and $\epsilon = 0.01$ (first mode).

10^{-6} m^4, $A = 5.0657 \times 10^{-3}$ m^2, $\rho = 6000$ kg/m^3, and $L = 4.0$ m.

In Fig. 2, the corrections to the frequency ratio $(\omega - \omega_0)/\omega_0$ (curves 1–4) and to the internal force amplitude ratio $(S - S_0)/S_0$ (curves 1^*–4^*) in terms of the value of nondimensional spring constant β are presented for various values of the amplitude parameter.

Figure 3 illustrates the effect of the amplitude parameter ϵ on the value of ω/ω_0 (curves 1–4) and S/S_0 (curves 1^*–4^*). The results of the calculation concerning variations of internal force S vs the value of external force P for various values of eccentricity e are gathered in Figs. 4a and 4b, whereas the change of nonlinear frequency is shown in Figs. 5a and 5b.

For $e = 0$ and force $P_c = \pi^2 EJ/L^2\,(1 + \beta)$, the frequency ω_0 equals zero. The correction factor of the frequency ω_2 calculated for the data assumed is as follows:

For $\beta = 0.010$:

$$\omega_2 = 108.5, \qquad \frac{\epsilon}{\sqrt{2}}\,\omega_2 = 0.787, \qquad \epsilon = 0.01$$

For $\beta = 0.001$:

$$\omega_2 = 34.5, \qquad \frac{\epsilon}{\sqrt{2}}\,\omega_2 = 0.24, \qquad \epsilon = 0.01$$

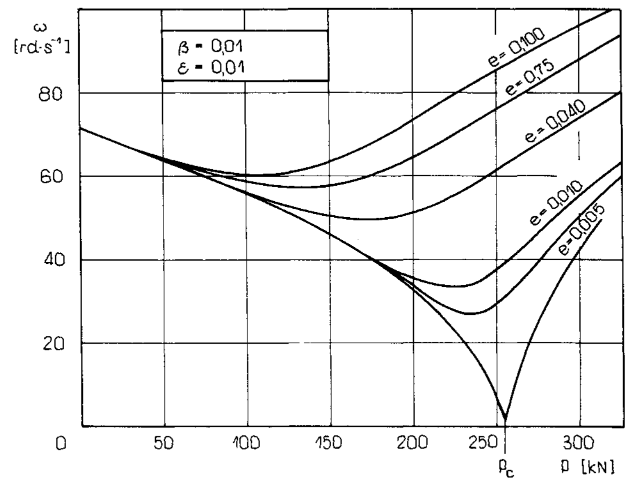

Fig. 5a Basic nonlinear frequency of the structure vs loading force P for various e at $\beta = 0.01$ and $\epsilon = 0.01$ (first mode).

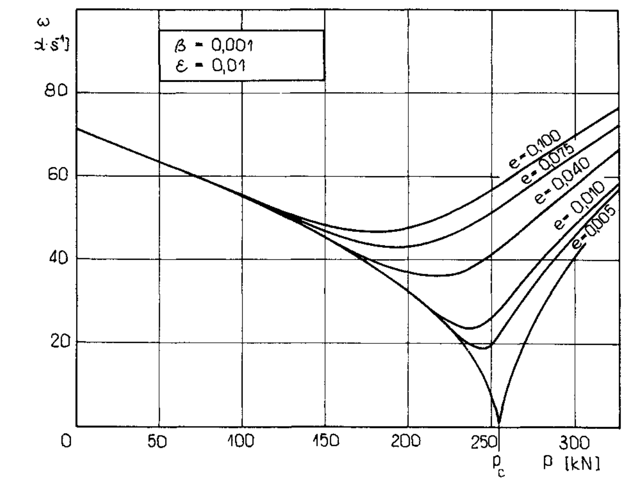

Fig. 5b Basic nonlinear frequency of the structure vs loading force P for various e at $\beta = 0.001$ and $\epsilon = 0.01$ (first mode).

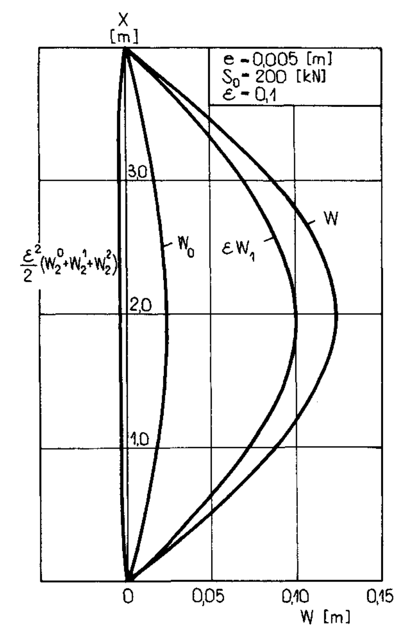

Fig. 6 First mode of vibration and components of beam axis deflection.

Table 1 Nondimensional mode shape

ξ	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\delta(\xi)$	0.156	0.308	0.453	0.587	0.707	0.805	0.891	0.951	0.988	1.000

Table 1 shows the nondimensional mode shape $\delta(\xi) = \{\epsilon w_1^1(\xi) + (\epsilon^2/2)[w_2^0(\xi) + w_2^2(\xi)]\}/\{\epsilon w_1^1(1/2) + (\epsilon^2/2)[w_2^0(1/2) + w_2^2(1/2)]\}$ calculated for $e = 0.005$, 0.050, 0.100, and $\epsilon = 0.10$ and $\epsilon = 0.01$. In Fig. 6, the first mode of vibration is shown.

Conclusions

The solution $w_1(\xi,\tau)$ is determined by the linear differential-integral equation, in which the term including the initial deflection w_0 (ξ) resulting from the system imperfection exists. Therefore, the first approximation w_0 depends also upon the initial deflection.

With the increase in the spring stiffness, the values of correction decrease both for nonlinear free vibration frequency and values of internal forces S (cf. Fig. 2). The increase in the value of the amplitude parameter results in the increase of the value of frequency ω and the decrease of amplitude of the internal force S (cf. Fig. 3).

A significant influence of the initial imperfection on the nonlinear frequencies ω and internal forces is also observed. However, this effect decreases with the increase in the value of spring constant (cf. Figs. 4 and 5).

For geometrically nonlinear simply supported beams with an initial imperfection, the change of the mode shape does not occur with the increase in the amplitude of vibration. The mode shape of vibration does not exhibit such a change with the increase in the initial imperfection.

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